Economics 2010-100 University of Colorado
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Mathematics Review

1. Graphing Lines

In this section we will help you understand how to graph lines on a coordinate plane. Most pre‐algebra courses cover how to graph simple equations such as the following:
y = 2x + 1

In math, a line is defined to be of infinite length and consisting of at least 2 points. All lines are straight (a line is straight ‐ a curve is curved).

When you need to graph an equation such as $y = -\frac{1}{2}x + 2$, the only thing you need to be especially wary of is the fraction. Since a line consists of two or more points, all you need to do is find two or more ordered pairs that solve the equation. The easiest way to do this is to draw a table such as the following and fill it in:

X	Y
0	
2	
4	

You plug the x‐values into the equation and find the y‐values. That gives you ordered pairs that you can graph on the coordinate plane and then "connect" into a line.

Example

1. Graph: $y = -\frac{1}{2}x + 2$

Solution: Begin by making a table (choose convenient values for x).

X	-2	0	2
Y			

Now plug the x – values into the original equation and find the values for y.

$$y = -\frac{1}{2}(-2) + 2$$
$$y = 3$$

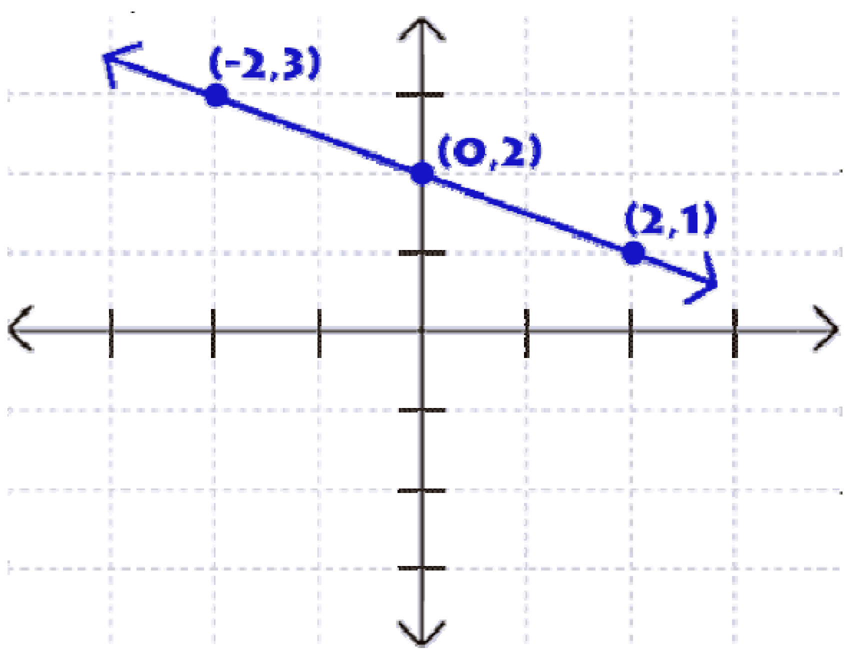
$$y = -\frac{1}{2}(0) + 2$$
$$y = 2$$

$$y = -\frac{1}{2}(2) + 2$$
$$y = 1$$

Complete the table.

X	-2	0	2
Y	3	2	1

Now graph the points and draw a line by "connecting the dots."
(Aren't you overwhelmed by all this fun?)
Here's what it looks like:



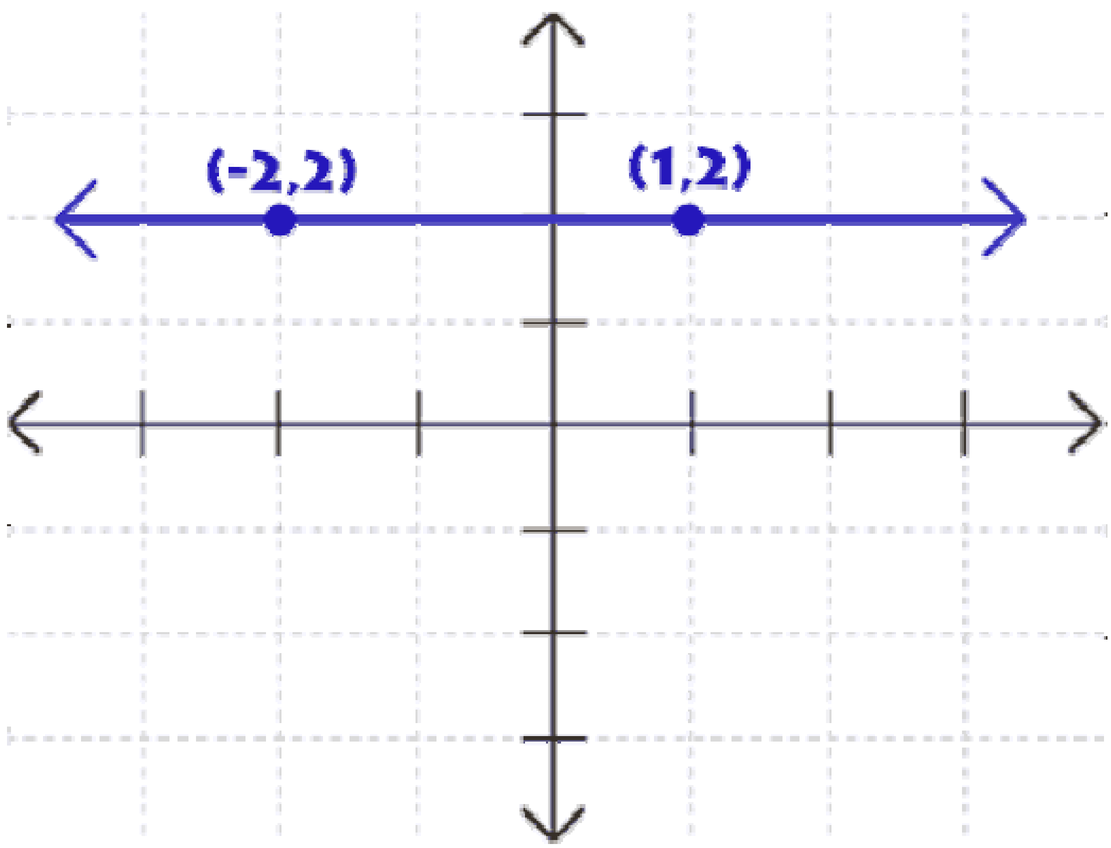
The most confusing types of lines are lines that are either horizontal or vertical. These are lines that are representative of an equation that has either an x variable or a y variable, but not both.

An equation such as y = 2 says that no matter what you plug in for y, you get 2.
An equation such as x = 4 has two things to keep in mind. First of all, it has no slope! This is because it is vertical. (With a horizontal line, the slope is zero, but with a vertical line, the slope is undefined, so the line therefore has no slope.) The other thing to remember is that no matter what you plug in for x, you'll get 4.

Example

2. Graph y = 2

Solution: This equation indicates that all the y coordinates to be graphed are 2.
Pick any two ordered pairs with 2 as the y coordinate and graph.



2. How to find the Slope

When graphed, lines slope from left to right. However, some slope upward and others slope downward. Some are really steep, while others have a gentle slope. The slope of a line is defined as the change in y over the change in x, or the rise over the run.

This can be explained with a formula: $(y_2 - y_1)/(x_2 - x_1)$. To find the slope, you pick any two points on the line and find the change in y, and then divide it by the change in x.

Example

1. Problem: The points (1,2) and (3,6) are on a line. Find the line's slope.
   Solution: Plug the given points into the slope formula.

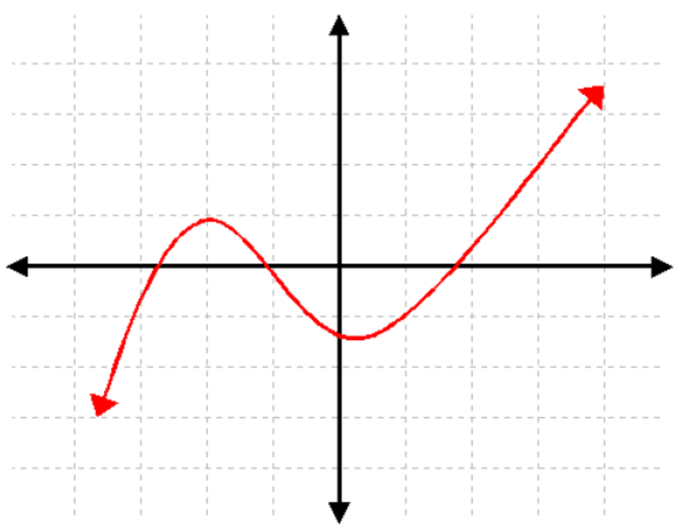
$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)} \quad \rightarrow \quad m = \frac{(6-2)}{(3-1)} = 2$$

After simplification, m = 2

3. About Functions

A function is a relation (usually an equation) in which no two ordered pairs have the same x – coordinate when graphed.

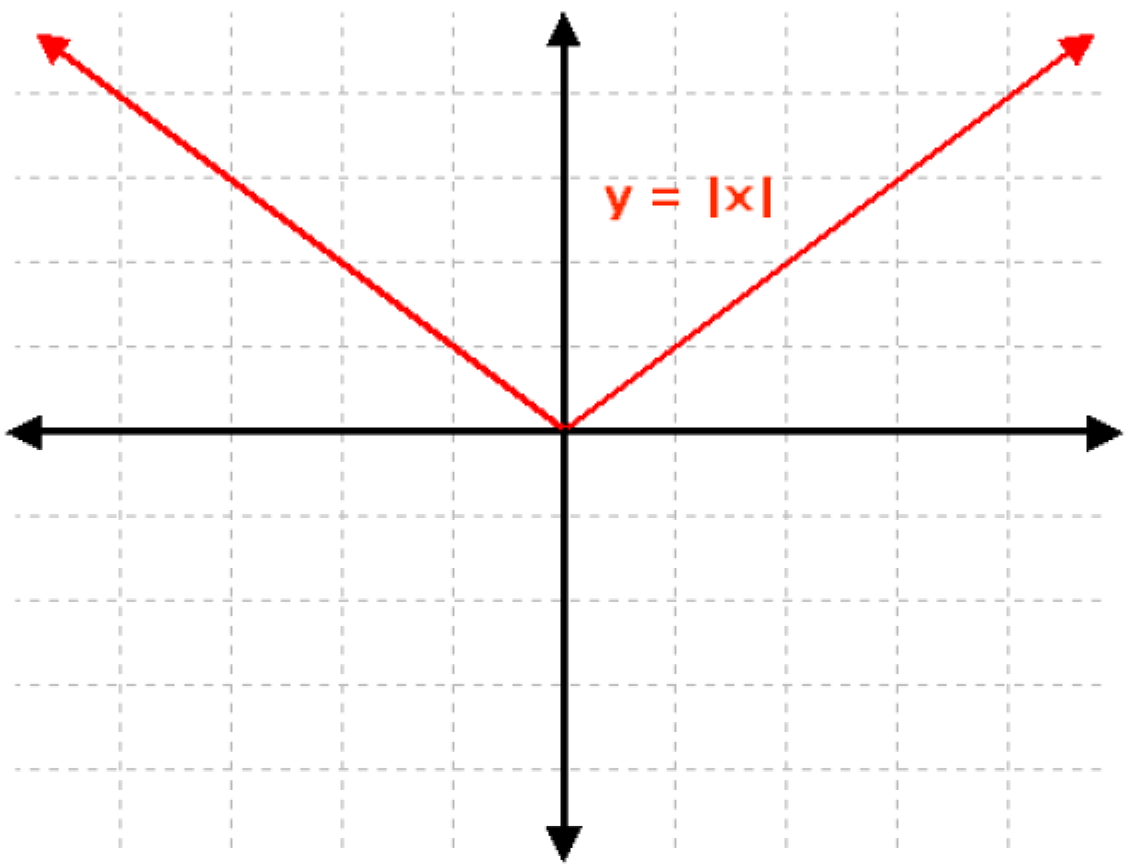
One way to tell if a graph is a function is the vertical line test, which says if it is possible for a vertical line to meet a graph more than once; the graph is not a function. The figure below is an example of a function.



Functions are usually denoted by letters such as f or g. If the first coordinate of an ordered pair is represented by x, the second coordinate (the y coordinate) can be represented by f(x). In the figure below, f(1) = -1 and f(3) = 2.

When a function is an equation, the domain is the set of numbers that are replacements for x that give a value for f(x) that is on the graph. Sometimes, certain replacements do not work, such as 0 in the following function: f(x) = 4/x (you cannot divide by 0). In that case, the domain is said to be x > 0.

There are a couple of special functions whose graphs you should have memorized because they are sometimes hard to graph. They are the absolute value function (below).



Example

1. Problem: solve $|5x - 4| = 11$

Solution: Use the theorem stated above to rewrite the equation.

$|X| = b$

$X = 5x - 4$ and $b = 11$

$5x - 4 = 11$ / $5x - 4 = -11$

$5x = 15$ / $5x = -7$

$x = 3$ / $x = -(7/5)$

4. Solving systems of equations

Solving systems of equations graphically is one of the easiest ways to solve systems of simple equations (it's usually not very practical for complex equations such as hyperbolas or circles). However, it is usually covered in elementary algebra (Algebra I) courses.

Another way to solve systems of equations is by substitution. In this method, you solve on equation for one variable, then you substitute that solution in the other equation, and solve.

Example

1. Problem: Solve the following system:

$x + y = 11$

$3x - y = 5$

Solution: Solve the first equation for y. (you could solve for x – it doesn't matter)

$y = 11 - x$

Now, substitute 11 - x for y in the second equation.

This gives the equation one variable, which earlier algebra work has taught you how to do.

$3x - (11 - x) = 5$

$3x - 11 + x = 5$

$4x = 16$ $x = 4$

Now, substitute 4 for x in either equation and solve for y.

(We use the first equation below.)

$4 + y = 11$

y = 7

The solution is the ordered pair, (4, 7).

The last method, addition, is probably the most complicated, but is necessary when dealing with more complex systems, such as systems with three or more variables. The idea behind the addition method is to replace an equation with a combination of the equations in the system. To obtain such a combination, you multiply each equation by a constant and add. You choose the constants so that the resulting coefficient of one of the variables will be 0.

Example

2. Problem: Solve the following system:

$5x + 3y = 7$

$3x - 5y = -23$

Solution: Multiply the second equation by 5 to make the x coefficient a multiple of 5.

$15x - 25y = -115$

Next, multiply the first equation by -3 and add it to the second equation.

This gets rid of the x-term.

$-15x - 9y = -21$

$15x - 25y = -115$

$-34y = -136$

Now, solve the second equation for y.

Then substitute the result into the first equation and solve for x.

$-34y = -136$

$y = 4$

$5x + 3(4) = 7$

$5x + 12 = 7$

$5x = -5 \qquad x = -1$

The solution is the ordered pair, (-1, 4).